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II. Solution by ARTEMAS MARTIN, LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C.

Let ax, ay, and az denote the numbers required. Then must $a^4x^4+a^4y^4+a^4z^4=\square$, or, rejecting the square factor a^4, we must make $x^4+y^4+z^4=\square=(x^2-y^2+z^2)^2$ say; whence $x^2z^2=x^2y^2+y^2z^2=y^2(x^2+z^2)$. Assume $x=p^2-q^2$ and $z=2pq$, and we have $xz=y(p^2+q^2)$, which gives $y=\dfrac{xz}{p^2+q^2}=\dfrac{2pq(p^2-q^2)}{p^2+q^2}$. Now take $a=p^2+q^2$ and we have $ax=p^4-q^4$, $ay=2pq(p^2-q^2)$, $az=2pq(p^2+q^2)$.

$$\therefore\ (p^4-q^4)^4+[2pq(p^2-q^2)]^4+[2pq(p^2+q^2)]^4=[(p^4-q^4)^2+12p^4q^4]^2=[(p^2+q^2)^4-4p^2q^2(p^2-q^2)^2]^2.$$

Take $p=2$, $q=1$ and we find the numbers 12, 15 and 20, the sum of whose 4th powers$=(481)^2$. These are the *smallest* numbers answering the problem, but an infinite number of other answers may be found by varying the values of p and q.

Also solved by *H. W. DRAUGHON, A. L. FOOTE, F. P. MATZ*, and *G. B. M. ZEER.*

10. Proposed by L. B. HAYWARD, Bingham, Ohio.

Find two numbers, such that each of them and also their sum and their difference when increased by unity shall all be square numbers.

I. Solution by JOSIAH H. DRUMMOND, LL. D., Portland, Maine.

The leaving out of commas leaves one in doubt whether each of the numbers *plus one*, or simply each of the numbers, must be a square. The more probable construction requires $x+1$ (1), $y+1$ (2), and $x\pm y+1$ (3) to be squares. Take $x+1=m^2+n^2$ and $y=2mn$ and the two conditions of (3) are met. But m^2+n^2 must be a square: this happens when $m=p^2-q^2$ and $n=2pq$. It now remains to make $2mn+1$, or $4pq(p^2-q^2)+1=\square=$(say) $(2pq\pm1)^2$. Reducing we have $p^2-qp=(q^2\pm1)$, from which $p=\dfrac{q\pm\sqrt{5q^2\pm4}}{2}$. $5q^2+4=\square=(sq\pm2)^2$; then $q=\pm\dfrac{4s}{s^2-5}$. We readily find $p=\dfrac{\pm2s\pm(s^2+5)}{s^2-5}$.

$s=1$, $q=\pm1$; $p=\pm1$, or ±2, $q=\pm1$, $p=\pm2$, $m=3$, and $n=4$ and $y=24$, and $x=24$. $s=2$, $q=\pm8$, $p=\pm5$ or ±13; $m=39$ and $n=80$; or $m=105$ and $n=208$; $x=7920$ or 54288; $y=6240$ or 43680. $s=3$, $q=\pm3$, $p=\pm5$ or ±2; $x=1155$ or 168; $y=960$ or 120; these last values of x and y are the least integrals which I have obtained. The reduotion of $\sqrt{(5q^2-4)}=\square$ is quite interesting, but as it gives no values for x and y, different from those already obtained, I omit it.

II. Solution by A. L. FOOTE, C. E., Merrick, New York, and P. S, BERG, Apple Creek, Ohio.

Solving under the other construction, let x^2 and $2x$ be the required numbers; then the sum increased by unity is x^2+2x+1 which is a square; and their difference increased by unity is x^2-2x+1 which is also a square, and since x^2 is a square it only remains to make $2x$ a square, which it is when $x=2$. But this value of x makes the numbers the same.

Our next value of x is 8 and $x^2=64$, and $2x=16$, which numbers answer the conditions. The next value of x is 18 and the numbers are 324 and 36, and so on *ad infinitum.*

Also solved by *W. H. DRAUGHON, ARTEMAS MARTIN, F. P. MATZ, J. F. W. SCHEFFER, G. B. M. ZERR*, and *J. K. ELLWOOD*.

PROBLEMS.

13. Proposed by ARTEMAS MARTIN, LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C.

It is is required to find four numbers the sum of whose fourth powers is a square number.

14. Proposed by SYLVESTER ROBINS, North Branch Depot, New Jersey.

Find initial terms in each of three infinite series of prime, integral, rational, scalene triangles, where 9 shall be the base, and the other two sides of every term shall have a constant difference.

15. Problems, or Propositions by M. A. GRUBER, M. A , War Department, Washington. D. C.

(*a*) The *difference* of two *odd* squares is always divisible by 8. Corollary: Every odd square is of the form $8a+1$.

(*b*) The *sum* of two *odd* squares is two times an *odd* number.

Solutions to these problems should be received on or before November 1st.

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AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Kidder, Mo. All Contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

6. Proposed by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

Find the average length of all the diameters that can be drawn in a given ellipse.

II. Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland, and the PROPOSER.

By using the *complement* of the eccentric angle we deduce $OD=a\sin\phi$